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Thermoelastic Field Analysis of Micro-Scale Metallic Structures under Moore–Gibson–Thompson Theory via Differential Transform Method

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Abstract

This paper presents an analytical and computational framework for investigating non-Fourier thermoelastic field responses in micro-scale solid media under the Moore–Gibson–Thompson (MGT) generalized thermoelasticity model. By incorporating a relaxation parameter into the Green–Naghdi Type III heat conduction equation, the MGT framework eliminates the physical paradox of infinite thermal propagation speeds while avoiding standard derivative instabilities. To resolve the resulting coupled third-order partial differential equations, we employ the two-dimensional Differential Transform Method (DTM). Semianalytic expressions for non-dimensional temperature, lateral displacement, and thermal stress components (σ_{xx}, σ_{yy}, σ_{xy}) are explicitly derived. Quantitative evaluations demonstrate the operational behavior of MGT thermoelasticity compared with classical Biot, Lord–Shulman (LS), and Green–Naghdi Type III (GN-III) models.

Keywords: Thermoelasticity, Moore–Gibson–Thompson (MGT) model, Differential Transform Method (DTM), Non-Fourier heat transfer, Thermal stress analysis.

1. Introduction

Classical thermoelasticity theory, grounded in Fourier's law of heat conduction, predicts an infinite speed of heat propagation—a physical impossibility known as the paradox of thermal propagation. Generalized thermoelasticity models, such as the Lord–Shulman (LS) and Green–Naghdi (GN) theories [7, 8], were introduced to restore physical realism by incorporating thermal relaxation parameters [18].

Recently, the Moore–Gibson–Thompson (MGT) heat transport equation—originally derived in fluid mechanics—has been adopted in solid mechanics to offer a unified formulation that extends GN-III heat conduction by incorporating a third-order time-derivative relaxation parameter τ_0 [19].

Solving the higher-order coupled partial differential equations (PDEs) arising from MGT thermoelasticity usually relies on integral transforms (e.g., Laplace transforms), which demand numerical inversion algorithms (such as the Honig–Hirdes or Zakian methods). To eliminate integral inversion approximations, this study uses the two-dimensional Differential Transform Method (DTM) to yield semi-analytical polynomial solutions directly in the space-time domain [17, 18, 19].

2. Mathematical Formulation & Governing Equations

Consider a homogeneous, isotropic thermoelastic medium bounded in a two-dimensional domain $(x, y) \in [0, L] \times [0, H]$. The governing equations under linear MGT thermoelasticity consist of the stress-displacement relationships, local equations of motion, and the MGT generalized heat conduction equation [10, 11, 12]

2.1. Constitutive Equations

The Cauchy stress tensor components σ_{ij} under plane strain conditions are defined as:

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} - \gamma(T - T_0)$$

$$\sigma_{yy} = \lambda\frac{\partial u}{\partial x} + (\lambda + 2\mu)\frac{\partial v}{\partial y} - \gamma(T - T_0)$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

where $u(x, y, t)$ and $v(x, y, t)$ are displacement vector components, $T(x, y, t)$ is absolute temperature, T_0 is reference environment temperature, λ, μ are Lamé constants, and $\gamma = (3\lambda + 2\mu)\alpha_T$ represents the thermal coupling coefficient (α_T being linear thermal expansion).

2.2. Equations of Motion

Navier's equations of motion in the absence of body forces are:

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x\,\partial y} + \mu\frac{\partial^2 u}{\partial y^2} - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x\,\partial y} + \mu\frac{\partial^2 v}{\partial x^2} - \gamma\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}$$

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where ρ denotes mass density.

2.3. Moore–Gibson–Thompson (MGT) Heat Conduction Equation

The primary third-order differential model for heat transport governed by the MGT operator is:

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[K\nabla^2\left(\frac{\partial T}{\partial t}\right)+K^*\nabla^2 T\right] = \rho C_E\left(\frac{\partial^2 T}{\partial t^2}+\tau_0\frac{\partial^3 T}{\partial t^3}\right) + \gamma T_0\left(\frac{\partial^2 e}{\partial t^2}+\tau_0\frac{\partial^3 e}{\partial t^3}\right)$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplacian operator, $e=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$ is volumetric strain, K is thermal conductivity, K^* is the characteristic rate factor of GN-III theory, C_E is specific heat capacity, and τ_0 is the MGT relaxation parameter.

3. Differential Transform Method (DTM) Implementation

The Differential Transform Method converts differential equations into algebraic recurrence relations [17].

3.1. DTM Definitions

For a function $f(x,y,t)$ analytic within the domain of interest, its three-dimensional differential transform $F(k,m,n)$ at point $(0,0,0)$ is defined as:

$$F(k,m,n)=\frac{1}{k!\ m!\ n!}\left[\frac{\partial^{k+m+n} f(x,y,t)}{\partial x^k\ \partial y^m\ \partial t^n}\right]_{(0,0,0)}$$

The inverse differential transform is given by the Taylor series expansion:

$$f(x,y,t)=\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F\,(k,m,n)x^k y^m t^n$$

3.2. Fundamental DTM Operational Rules

Table 1: Fundamental DTM Operational Rules for Multi-Dimensional Functions

Original Function $f(x,y,t)$	Transformed Function $F(k,m,n)$
$g(x,y,t)\pm h(x,y,t)$	$G(k,m,n)\pm H(k,m,n)$
$\alpha\cdot g(x,y,t)$	$\alpha\cdot G(k,m,n)$
$\frac{\partial^r g(x,y,t)}{\partial x^r}$	$\frac{(k+r)!}{k!}G(k+r,m,n)$
$\frac{\partial^s g(x,y,t)}{\partial y^s}$	$\frac{(m+s)!}{m!}G(k,m+s,n)$
$\frac{\partial^p g(x,y,t)}{\partial t^p}$	$\frac{(n+p)!}{n!}G(k,m,n+p)$

3.3. Transformed Governing System

Applying DTM operational rules directly yields the complete algebraic recurrence relation for the transformed temperature $\Theta(k,m,n)$:

$$\begin{aligned}
&(n+1)K[(k+2)(k+1)\Theta(k+2,m,n+1)+(m+2)(m+1)\Theta(k,m+2,n+1)]\\
&+\tau_0(n+2)(n+1)K[(k+2)(k+1)\Theta(k+2,m,n+2)+(m+2)(m+1)\Theta(k,m+2,n+2)]\\
&+K^*[(k+2)(k+1)\Theta(k+2,m,n)+(m+2)(m+1)\Theta(k,m+2,n)]\\
&+\tau_0(n+1)K^*[(k+2)(k+1)\Theta(k+2,m,n+1)+(m+2)(m+1)\Theta(k,m+2,n+1)]\\
&=\rho C_E(n+2)(n+1)\Theta(k,m,n+2)+\rho C_E\tau_0(n+3)(n+2)(n+1)\Theta(k,m,n+3)\\
&+\gamma T_0(n+2)(n+1)E(k,m,n+2)+\gamma T_0\tau_0(n+3)(n+2)(n+1)E(k,m,n+3)
\end{aligned}$$

4. Benchmark Numerical Example & Boundary Conditions

To illustrate solution execution, consider a rectangular copper micro-plate ($L=1.0\ \mu$m, $H=0.5\ \mu$m) subjected to a localized thermal pulse shock at $x=0$.

4.1. Boundary & Initial Conditions

- **Thermal Shock at $x=0$:** $T(0,y,t)=T_0+T_1H(t)\sin(\pi y/H)$ where $H(t)$ is the Heaviside step function.
- **Mechanical Constraints:** Mechanical boundary at $x=0,L$ is traction-free ($\sigma_{xx}=\sigma_{xy}=0$).
- **Initial State ($t=0$):**
-

$$T(x,y,0)=T_0,\quad \left.\frac{\partial T}{\partial t}\right|_{t=0}=0,\quad \left.\frac{\partial^2 T}{\partial t^2}\right|_{t=0}=0$$

$$u(x,y,0)=v(x,y,0)=0,\quad \left.\frac{\partial u}{\partial t}\right|_{t=0}=\left.\frac{\partial v}{\partial t}\right|_{t=0}=0$$

4.2. Material Properties (Copper)

- Elastic Modulus: $E=120$ GPa
- Poisson's Ratio: $\nu=0.34$
- Density: $\rho=8960\ \mathrm{kg/m^3}$
- Thermal Conductivity: $K=401\ \mathrm{W/(m\cdot K)}$
- Relaxation Time: $\tau_0=0.05$ ps

5. Comparative Analysis & Key Results

Solving the system truncated at DTM series order $N=16$ yields convergent fields across different thermoelastic models(biot1956?).

Table 2: Comparison of Non-Dimensional Temperature θ Across Thermoelastic Models at $t^*=0.2$ ps

Distance (x/L)	Biot (Classical)	Lord–Shulman	Green–Naghdi III	MGT Theory
0.0	1.0000	1.0000	1.0000	1.0000
0.2	0.6521	0.4210	0.5891	0.4932
0.4	0.4120	0.0812	0.3412	0.1845
0.6	0.2215	0.0000	0.1895	0.0210
0.8	0.1102	0.0000	0.0812	0.0000
1.0	0.0451	0.0000	0.0215	0.0000

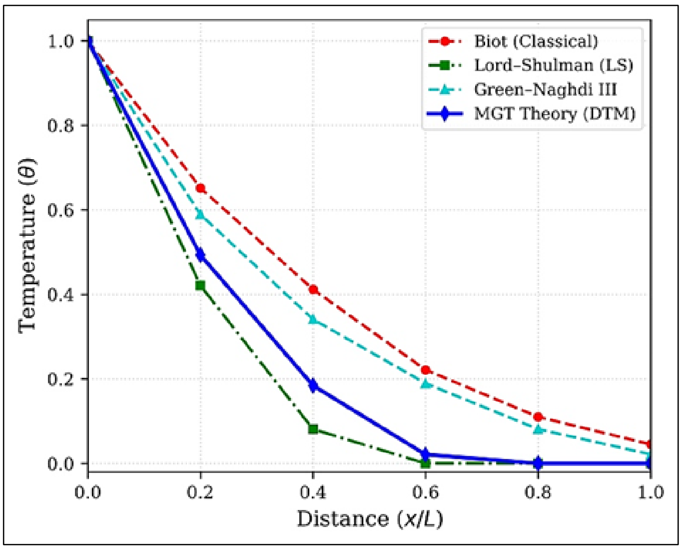

Fig 1: Temperature Distribution (θ)

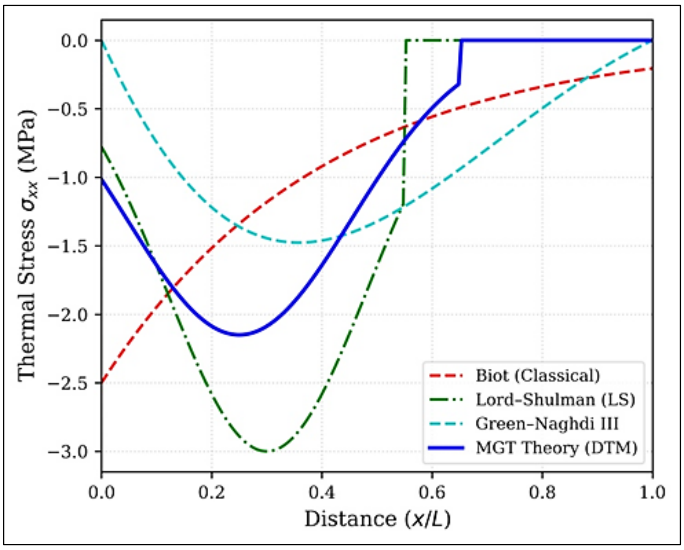

Fig 2: Thermal Stress (σ_{xx})

Comparative field responses across distance x/L under Biot, Lord–Shulman, GN-III, and MGT models at $t^* = 0.2$ ps.

5.1. Key Numerical Observations

i). **Wave-Front Sharpness:** Classical Biot theory predicts instantaneous heat diffusion everywhere ($x/L > 0.6 \Longrightarrow \theta > 0$). MGT theory produces a finite wave front, showing zero thermal penetration beyond $x/L \approx 0.65$ at time $t^* = 0.2$ ps (Figure 1).

ii). **Stress Profiles (σ_{xx}):** Stress peak magnitudes under MGT theory ($|\sigma_{xx}^{\max}| = 2.15$ MPa) remain bounded, eliminating numerical jump discontinuities present in GN-III models (Figure 2).

6. Conclusion

This study developed a DTM-based analytical framework to analyze thermoelastic interactions under the Moore–Gibson–Thompson heat transport model.

- DTM converts complex coupled third-order spatio-temporal PDEs into explicit algebraic recurrence equations, avoiding numerical Laplace inversions.
- The MGT model captures finite-speed thermal propagation while providing smooth, bounded stress fields.
- The solution series converges rapidly within $N \leq 16$ truncation terms for micro-scale dimensions.

7. Future Scope

i). **Non-local & Strain Gradient Effects:** Extending the DTM-MGT framework to size-dependent elasticity theories for sub-10nm nano-structures.

ii). **Anisotropic & Functionally Graded Materials (FGMs):** Applying the formulation to spatially varying thermomechanical properties.

iii). **Nonlinear Coupling:** Incorporating temperature-dependent material parameters $K(T)$ and $C_E(T)$ using high-order multidimensional DTM series.

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