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A Study on $\underline{P}$I Closed Set in Ideal Topological Space

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Abstract

The main objective of this article is to introduce a new class of genearalised sets namely g $\bar{p}$ I –closed set in topological space with respect to an ideal and investigate the relation between the set with other sets in ideal topological space and some of its basic topological properties and characterization. Also we analyse relationship of these sets with some of the know closed sets are studied.

Keywords: Children rights convention (CRC), national commission on the right of child (NCRC), child rights

Introduction

Topological space is a set endowed with a structure called a topology. Ideals in topological space have been considered since 1930. This topic has won its importance by the paper of the paper of Vaidyanathaswamy [19]. This concepts has been extended to the setting I-continuity of functions.

Jankovic and Hamlett [6, 7] introduced the notion of I-open sets in topological spaces. Abd EI-Monsef *et al.* [3] further investigate I-open sets and I-continuous functions. Dontchev [7] introduced the notation of preI-open sets and obtained a decomposition of I-continuity. In addition to this, Caksu Guler and Aslim [12] have introduced the notion of b I-sets and b I-continuous functions. An ideal on a set X is a non-empty collection of subsets of X with heredity property which is also closed under finite unions.

In this paper we introduce new class of generalized $\bar{P}$ I closed set in ideal topological space with respect to an ideal and their properties.

Basics Definitions

In this section we summarize the definitions and results which are needed in the sequel. By a space we always mean a topological space (X, τ) with no separation properties assumed. If A ⊆ X, cl(A) and int(A) denote the closure and interior of A in (X, τ) respectively. An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties.

i). A ∈ I and B ⊆ A imply B ∈ I

ii). A ∈ I and B ∈ I imply A ∪ B ∈ I.

A topological space with respect to an ideal is denoted by (X, τ, I). Given a topological space (X, τ) with an ideal I on X and if ℘ (X) is the set of all subsets of X, a set operator $(.)^*$: ℘ (X) → ℘ (X), called a local function of A with respect to I and τ is defined as follows: for A ⊆ X,

$A^*(I, \tau) = \{x \in X / A \cap U \in / I$, for every $U \in \tau(x)\}$, where

$\tau(x) = \{U \in \tau / x \in U\}$ [17]. Note that $cl^*(A) = A \cup A^*$ defines a Kuratowski operator for a topology $\tau^*(I)$ (also denoted by if there is no ambiguity), finer than τ. A basis β(I, τ) for $\tau^*(I)$ can be described as follows:

$\beta(I, \tau) = \{U \setminus I : U \in \tau$ and $I \in I\}$. β is not always a topology [13]. $cl^*(A)$ and $int^*(A)$ denote the closure and interior of A in (X, τ^*) respectively.

Definition 1.1. [22]**:** An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties.

a) A ∈ I and B ⊆ A imply B ∈ I

b) A ∈ I and B ∈ I imply A ∪ B ∈ I.

A topological space with respect to an ideal is denoted by (X, τ, I).

Lemma 1.2. [20]**:** If A satisfies topological space (X, τ) then

i). int(A) ⊆ int(cl(int(A))) ⊆ cl(int(A)) ⊆ cl(int(cl(A))) ⊆ cl(A).

ii). int(A) ⊆ int(cl(int(A))) ⊆ int(cl(A)) ⊆ cl(int(cl(A))) ⊆ cl(A).

Lemma 1.3.: If A is a subset of a topological space (X, τ) then

int(A) satisfies:[20]

a) int(A) ⊂ A.

b) int(int(A)) = int(A).

c) A is open iff int(A) = A.

Definition 1.4. [12]**:** A subset A of a topological space with respect to an ideal (X, τ, I) is called

i). I-open if $A \subseteq int(A^*)$.

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ii). [16] regular I-Open if $A = int(cl^*(A))$
iii). [6] Pre I-Open if $A \subseteq int(cl^*(A))$
iv). [9] Semi I-open if $A \subseteq cl^*(int(A))$
v). [9] αI-Open if $A \subseteq int(cl^*(int(A)))$
vi). [9] semi pre I-Open if $A \subseteq cl(int(cl^*(A)))$
vii). [21] SI set if $cl^*(int(A)) = int(A)$.

Lemma 1.5. [22]**:** For any subset A of an ideal topological space (X, τ, I), the following result hold.

a) $sIcl(A) = A \cup int(cl^*(A))$
b) $pIcl(A) = A \cup cl^*(int(A))$
c) $spIcl(A) = A \cup int(cl^*(int(A)))$
d) $cl^*(int(A \cup B)) = cl^*(int(A)) \cup cl^*(int(B))$

Lemma 1.6. [21]**:** Let (X,τ,I) be a topological space with respect to an ideal and $A \subseteq X$. If $A \subseteq A^*$, then $A^* = cl(A) = cl^*(A)$.

Remark 1.7.: A is open if and only if int(A) = A and A is *-open if and only if $A = int^*(A)$.

Result

Generalised $\bar{p}$I closes set

Definition 2.1.: A subset A of a topological space with respect to an ideal (X, τ, I) is said to be generalised $\bar{p}$I-closed (briefly g $\bar{p}$ I-closed) if $pIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is bI-open.

Example 2.2.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d}}.

Theorem 2.3.: A Subset of a g $\bar{p}$ I-closed set is need not be g $\bar{p}$ I closed set

Example 2.4.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d}}. In this ideal space, the set {b,c,d} is g $\bar{p}$I-closed set but the subset {d} is not g $\bar{p}$ I-closed set.

Theorem 2.5.: Every Closed set is g $\bar{p}$ I-closed set

Proof: Let A be a closed set in X. Let $A \subseteq U$ and U be bI-open set. Since A is closed we have A = cl(A), $cl(A) \subseteq U$.
But $pIcl(A) \subseteq cl(A) \subseteq U$. Therefore A is g $\bar{p}$ I-closed set.

Remark 2.6.: The following example shows that the converse of the above theorem is not true.

Example 2.7.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d}}. In this ideal space, the set {b,c,d} is g $\bar{p}$I-closed set but the subset {d} is not in closed set.

Theorem 2.8.: Every Pre I-closed set is g $\bar{p}$I-closed set

Proof: Let A be a pre I-closed set in X. We know that pre I-closure of A is the smallest pre I-closed containing A. Therefore $pIcl(A) \subseteq A$. Suppose $A \subseteq U$ and U is bI-open. Then $pIcl(A) \subseteq U$ and U is bI-open. Hence A is g $\bar{p}$I-closed set

Remark 2.9.: The following example shows that the converse of the above theorem is not true.

Example 2.10.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}} I = {φ, {c}, {d}, {c, d}}. In this ideal space, the set {a,b,d} is g $\bar{p}$I-closed set but not Pre I-closed set.

Theorem 2.11.: Every SI set is g $\bar{p}$I-closed set

Proof: Let A be a SI-closed subset of X. Let $A \subseteq U$ and U be bI-open. Since A is SI-set we have $A = cl^*(int(A)) = int(A)$.
$A \subseteq U \Rightarrow int(A) \subseteq int(U) \subseteq U \Rightarrow cl^*(int(A)) \subseteq U \Rightarrow A \cup cl^*(int(A)) \subseteq A \cup U = U \Rightarrow pIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is bI-open. Hence A is g $\bar{p}$I-closed set.

Remark 2.12.: The following example shows that the converse of the above theorem is not true.

Example 2.13.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d}}. In this ideal space, the set {a,c,d} is g $\bar{p}$I-closed set but not SI set

Theorem 2.14.: Every rI-closed set is g $\bar{p}$I-closed set

Proof: Let A be a r I-closed subset of X. Let $A \subseteq U$ and U be bI-open. Since A is rI-closed we have $A = cl^*(int(A)) \Rightarrow cl^*(int(A)) \subseteq U$ and U be bI-open $\Rightarrow A \cup cl^*(int(A)) \subseteq A \cup U$ and U be bI-open.
By Lemma 1.5 (ii) we have $pIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is bI-open. Hence A is g $\bar{p}$I-closed set.

Remark 2.15.: The following example shows that the converse of the above theorem is not true.

Example 2.16.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d},the set {a,b} is g $\bar{p}$I-closed set but not rI-closed

Theorem 2.17.: Every α I-closed set is g $\bar{p}$I-closed set

Proof: Let A be a α I-closed set in X. We know that every α I-closed set is preI-closed set. By theorem 2.8, A is g $\bar{p}$ I-closed set

Remark 2.18.: The following example shows that the converse of the above theorem is not true.

Example 2.19.: Consider X = {a,b,c,d} with τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d}},the set {a,d} is g $\bar{p}$I-closed set but not α I-closed set.

Remark 2.20.: Every semiI-closed set is g $\bar{p}$I-closed set but converse is not true

Example 2.21.: Consider X = {a,b,c,d}, τ = {X, φ, {a, c}, {d}, {a, c,d}}. I = {φ, {c}, {d}, {c, d}}, In this ideal space, the set {a,c,d} is g $\bar{p}$I-closed set but not Semi I-closed set

Theorem 2.22.: Every semipre I-closed set is g $\bar{p}$I-closed set

Proof: Let A be a semipre I-closed set in X. We know that every semipre I-closed set is pre I-closed set. By theorem 2.8, A is g $\bar{p}$ I-closed set

Remark 2.23.: The following example shows that the converse of the above theorem is not true.

Example 2.24.: Consider X = {a,b,c,d}, τ = {X, φ, {a, c}, {d}, {a, c,d}}.
I = {φ, {c}, {d}, {c, d}}, the set {a,b,c} is g $\bar{p}$I-closed set but not spI-closed set.

Remark 2.25.: g $\bar{p}$I-closed set and preI-closed sets are independent to each other as seen from the following example

Example 2.26.: Let X = {a, b, c, d} τ = {X, φ, {a, c}, {d}, {a, c,d}}. Let I = {φ, {b}}. Clearly the set{a,b,d} which is g $\bar{p}$I-closed set but not preI-closed set.

Theorem 2.27.: If A and B are g $\bar{p}$I-closed sets in (X, τ, I) then A∪B is also g $\bar{p}$I-closed set

Proof: Let A ∪ B ⊆ U where U is bI-open in X. Then A ⊆ U and B ⊆ U. Since A and B are g $\bar{p}$I-closed set, then pIcl(A) ⊆ U and pIcl(B) ⊆ U and so pIcl(A) ∪ pIcl(B) ⊆ U. Since A ⊆ U and B ⊆ U
Hence A ∪ B is g $\bar{p}$I-closed set.

Remark 2.28.: The intersection of g $\bar{p}$I-closed sets need not be g $\bar{p}$I-closed set as shown in the following example

Example 2.29.: Let X = {a, b, c, d}, τ = {X, φ, {a, c}, {d}, {a, c, d}}. Let I = {φ, {b}}. Clearly the set{a,b,d} which is g $\bar{p}$I-closed set but not preI-closed set. Let A = {a,b,c} and B = { a,c,d} A ∩ B = {a,c} not in g $\bar{p}$I-closed set.

Theorem 2.30.: Let A be an g $\bar{p}$I-closed set of (X, τ, I) and A ⊂ B ⊂ pIcl(A), then B is g $\bar{p}$I-closed set in X.

Proof: Let B ⊂ U, where U is bI-open in X. ⇒ B ⊆ pIcl(A) ⇒ pIcl(B) ⊂ pIcl(pIcl(A)) = pIcl(A). Since A is g $\bar{p}$ I-closed set and A ⊂ U, pIcl(A) ⊆ U. Therefore pIcl(B) ⊆ U ⇒ B is g $\bar{p}$I-closed set.

Theorem 2.31.: If A is preI-closed and cl * (int (A)) is open then A is g $\bar{p}$I-closed set

Proof: Let A ⊆ U and U be bI-open.
i). Since A is preI-closed and by Lemma 2.8
ii). A ∪ cl * (int (A)) ⊆ U. ie) pIcl(A) ⊆ U whenever A ⊆ U and U is bI-open. Hence A is g $\bar{p}$I-closed set.

Theorem 2.32.: For every point x of a space X, X \ {x} is g $\bar{p}$I-closed set or bI-open

Proof: Suppose X \ {x} is not I-open. Then X is the only bI-open set containing X \ {x}. ⇒ pIcl(X \ {x}) ⊆ X. Hence X \ {x} is g $\bar{p}$I-closed set in x.

Theorem 2.33.: If A is g $\bar{p}$I-closed set then pIcl(A) \ A does not contain a non-empty bI-closed set

Proof: Suppose A is g $\bar{p}$I-closed. Let F be a bI-closed subset of pIcl(A) \ A. But A is g $\bar{p}$I-closed set and since X \ F is bI-open, we have pIcl(A) ⊆ X \ F. Therefore F ⊂ X \ pIcl(A). Since F ⊆ pIcl(A), we have F ⊆ (X \ pIcl(A) ∩ pIcl(A) = Φ). ⇒ F = φ.

Therefore pIcl(A) \ A does not contain non-empty bI-closed.

Theorem 2.34.: If A is g $\bar{p}$I-closed set and if A ⊆ B ⊆ pIcl(A) then
i). B is g $\bar{p}$I-closed set
ii). pIcl(B) \ B contains no non-empty g $\bar{p}$I-closed set

Proof:
i). Given A ⊆ B ⊆ pIcl(A). Then pIcl(A) = pIcl(B). Suppose that B⊆ U and U is bI-open. Since A is g $\bar{p}$I-closed and A ⊆ B ⊆ U, pIcl(A) ⊆ U we have pIcl(B) ⊆ U. Hence B is g $\bar{p}$I-closed set.
ii). The proof follows from Theorem 3.31.

References

1. Abd EI-Monsef ME, Mahmoud RA, Lashien EF. "On I open sets and I-continuous mappings", Kyungpook *Mathematical Journal*. 1992; 32(1):21-30.
2. Abd EI-Monsef ME, Mahmoud RA, Nasef AA. "Almost I-openness and almost I-continuity", *J Egyptian Matah. Soc*. 1999; 7(2):191-200.
3. Abd El-Monsef ME, Nasef AA, Radwan AE, Esmaeel RB. "On α-open sets with respect to an ideal", *Journal of Advanced Studies in Topology*. 2014; 5(3):1-9.
4. Balachandran K, Sundaram P, Maki H. "On generalized continuous maps in topological spaces", Mem. Fac. Sci. Kochi Univ.Ser A. Math. 1991; 12:5-13.
5. Crossley SG, Hildebrand SK. "Semi-topological properties", Fund. Math. 1972; 74:233-254.
6. Dontchev J. "Contra-continuous functions and strongly S-closed spaces", *Internat. Math. Sci*. 1996; 19:303-310.
7. Dontchev J, "Idealization of Ganster Reilly Decomposition Theorems", Math.GN/9901017,(5) Jan.1999.
8. Ekici E. "On RI open sets and A* sets in ideal topological spaces", Annals of the Univ., of Craiova, Mathematics and computer science series. 2011; 38(2):26-31.
9. Hatir E, Noiri T. "On decomposition of continuity via idealization", Acta Math. Hungar. 2002; 96(4):341-349.
10. Jain RC. "The role of regularly open sets in general topology Ph.D. thesis", Meerut University, Meerut, India, 1980.
11. Jamal M, Mustafa, "Contra semiI-continuous functions", *Hacettepe J Math*. Stat. 2010; 39(2):191-196.
12. Jankovic D, Hamlett TR. "Compatible extension of ideals", Boll. Un. mat. Ital. 1992; 6(3):453-465.
13. Jankovic D, Hamlett TR. "New topologies from old via ideals", Amer. Math. Monthly. 1990; 97:295-310.
14. Jethruth Thresa G, Murugesan S. "On α gsp I-closed sets in topological space with respect to an ideal", Proceeding of the International Conference on "Recent Trends in Applied Mathematics", Sri S. Ramasamy Naidu Memorial College, Sattur, January 2017, (173-181).
15. Jethruth Thresa G, Murugesan S. "On α gsp I-continuous function in topological space with respect to an ideal" *International Journal of Science Engineering and Technology Research*. 2017; 6(4):477-481.
16. Keskin A, Noiri T, Yuksel S. "Idealization of a decomposition theorem", Acta. Math. Hungar. 2004; 102:269-277.
17. Kuratowski K. "Topology", Academic Press New York, 1966, I.
18. Levine N. "Generalized closed sets in Topology", Rend. Circ. Mat. Palermo. 1970; 19(2):89-96.
19. Vaidhayanathaswamy. R Set topology, 2nd edi., Chelsea publishing, New York,1960.

20. Munkers James R. “Topology”, II edition, Pearson Education Singapore Pvt.Ltd. India branch, 2011.
21. Renuka Devi V, Sivaraj D, Tamizhchelvam T. “Properties of some *-dense in itself subsets”, IJMMS. 2004; 72:3989-3999.
22. Renuka Devi V, Sivaraj D, Tamizhchelvam T. “Studies on topological spaces with respect to an ideal”, Ph.d. Thesis. Manonmaniam Sundaranar University, Tirunelveli, 2007.