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A Note on Relative Ordered (m,n)-Hyperideals in Ordered Semihypergroups

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Abstract

In this paper, we study relative ordered (m, n)-hyperideals in ordered semihypergroups. We also study relative (m, 0)-hyperideals and relative (0, n)-hyperideals as well as characterize regular ordered semihypergroups, and obtain some results based on these relative hyperideals. We prove that the intersection of all relative ordered (m, n)-hyperideals of S containing s is a relative ordered (m, n)-hyperideal of S containing s. Suppose that (S,◦,≤) is an ordered semihypergroup, A ⊆ S and m,n are positive integers. We prove that if R(m,0) and L(0,n) be the set of all relative ordered (m,0)-hyperideals and the set of all relative ordered (0,n)-hyperideals of S, respectively. Then the following assertions are true: i) S is relative (m,0)-regular if and only if for all R ∈ R(m,0), R = (Rm ◦A]A. ii) S is relative (0,n)-regular if and only if for all L ∈ R(0,n), L = (A◦Ln]A. Furthermore, suppose that (S, ◦, ≤) is an ordered semihypergroup and m, n are non-negative integers. Let A ⊆ S. Suppose that A(m,n) is the set of all relative ordered (m, n)-hyperideals of S. Then, we have the following: S is (m,n)-regular ⇐⇒ ∀A ∈ A(m,n),A = (Am ◦A◦An]A.

Keywords: Ordered semihypergroup, regular ordered semihypergroup, relative ordered bi-hyperideal, relative ordered (m, n)-hyperideal

Introduction

1. Preliminaries

The concept of bi-ideal in semigroups was given by Good and Hughes [9] and the con-cept of generalized bi-ideal in rings was introduced by Szasz [5, 6], and thereafter in semigroups by Lajos [10].

Marty [4] introduced hyperstructures. In other algebraic structures, the composition of two elements is an element, whereas in algebraic hyperstructures, the composition of two elements is a set. This feature makes hyperstructures generalize other algebraic structures.

Abbasi *et al.* [7] studied ordered (m, n)-Γ-ideal in ordered-Γ-semigroups Khan [8] studied relative bi-ideals and relative quasi ideals in ordered semigroup. Recently, Basar *et al.* [1, 2, 3] studied relative hyperideals. A hyperstructure S is a nonvoid set equipped with an hyperoperation "◦" on S defined as follows:

$$\circ : S \times S \to P^*(S) \mid (x, y) \to (x \circ y)$$

And an operation "∗" on P∗(S) defined as follows:

$$* : P^*(S)\times P^*(S) \to P^*(S) \mid (X,Y) \to X * Y$$

Such that

$$X * Y = (x \circ y)$$
$$(x, y) \in X\times Y$$

for any X,Y ∈ P*(S), where P*(S) denotes the nonempty subsets of H. A hyperoper-ation "◦" on S gives rise to an operation "∗" on P*(S). Conversely, an operation "∗" on P*(S) gives rise to a hyperoperation "◦" on S, defined as follows: x◦y = {x}∗{y}. Therefore, a hypersemigroup (S,◦,∗) can be identified by (S,◦) because of the inter-dependency of the operation " ∗ " and the hyperoperation " ◦ ". Clearly, we have X ⊆ Y ⇒ X ∗D ⊆ Y ∗D, D∗X ⊆ D∗Y for any X, Y, D ∈ P*(S) and S ∗S ⊆ H. For a subset X of an hypersemigroup S, we define by (X] the subset of S as follows:

$$(X] = \{s \in S :\mid s \le x \text{ for some } x \in X\}.$$

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If " $\leq$ " is an order relation on a hypersemigroup S, we define the order relation " $\preceq$ " on $P^*(S)$ as follows:

$$\preceq := \{(X, Y) \mid \forall x \in X\ \exists\ y \in Y \text{ such that } x \leq y\}.$$

Therefore, for X, Y ∈ $P^*(S)$, we denote X ⪯ Y if for every x ∈ X, there exists y ∈ Y such that x ≤ y. This is indeed, a reflexive and transitive relation on $P^*(S)$.

A hyperstructure (S,∘) is called a semihypergroup if for all x,y,z ∈ S, (x ∘ y) ∘ z = x∘(y ∘z), i. e., m∘z = x∘n. m∈x∘y n∈y∘z

A nonempty subset A of a semihypergroup (S,∘) is called a subsemihypergroup of S if A∗A ⊆ A. An semihypergroup (S,∘) equipped with a partial order " ≤ " on S that is compatible with semihypergroup operation " ⪯ " such that for all x, y, z ∈ S,

$$x \leq y \Rightarrow z \circ x \preceq z \circ y \text{ and } x \circ z \preceq y \circ z,$$

ia called an ordered semihypergroup.

For subsets P, Q of an ordered semihypergroup S, the product set P ∘ Q of the tuple (P,Q) relative to S is defined as P ∘ Q = {p ∘ q | p ∈ P, q ∈ Q} and for P ⊆ S, the product set P ∘P relative to S is defined as P2 = P ∘P.

We denote $(\{s\}]_A$ by (s]A for s ∈ A ⊆ S.
Let A ⊆ S. Then, for a non-negative integer n, the power of A^n = A∘A∘A⋯, where A appears n times. It is to be noted that the exponent is suppressed when n = 0. Thus, we have $A^0 \circ S = S = S \circ A^0$.

We denote the ordered semihypergroup (S,∘,≤) by S unless otherwise specified. Let A and B be two nonempty subsets of S. Then, we have the following:

1. (A]∘(B] ⊆ (A∘B];
2. A ⊆ B ⇒ (A] ⊆ (B];

Definition 1.1. Suppose that B is a sub-semihypergroup (sub-semihypergroup) (resp. nonempty subset) of an ordered semihypergroup S and A ⊆ S. Then, B is called a relative (resp. generalized) (m, n)-hyperideal of S if
i) $B^m \circ A \circ B^n \subseteq B$,
ii) For b ∈ B, s ∈ A, s ≤ b ⇒ s ∈ B.

Note that in the above Definition 1.1, if m = n = 1, then B is called a (generalized) relative bi-hyperideal of S.

Definition 1.2. Suppose that (S,∘,≤) is an ordered semihypergroup and m,n are nonnegative integers. Let A ⊆ S. Then, S is called relative (m, n)-regular if for any s ∈ A, there exists x ∈ A such that $s \leq s^m \circ x \circ s^n$ for s1, s2 ∈ A. In another equivalent form, it defines that (S,∘,≤) is relative (m, n)-regular if $s \in (s^m \circ A \circ s^n]A$ for all s ∈ A.

2. Characterization of Ordered Semihypergroups by Relative Ordered (m, n)-Hyperideals

In this section, we provide some new characterization of ordered semihypergroups by the properties of relative ordered (m, n)-hyperideals. We begin by the following:

Proposition 2.1: Suppose that (S,∘,≤) is an ordered semihypergroup and s ∈ A ⊆ S. Furthermore, suppose that m and n are non-negative integers. Then the intersection of all relative ordered (m, n)-hyperideals of S containing s is a relative ordered (m,n)-hyperideal of S containing s.

Proof: Suppose that A ⊆ S, and {Bi : i ∈ I} is the set of all relative ordered (m, n)-hyperideals of S containing s ∈ A. Clearly, i∈I Ai is a sub-semihypergroup of S containing s ∈ A. Let j ∈ I. Since i∈I Ai ⊆ Aj, we receive the following:
(\Bi)m ∘A∘(\Bi)n ⊆ Am ∘A∘Bn ⊆ Bj.
Thus, we obtain the following:
(\Bi) ∘A∘(\Bi) ⊆ \Bi.
i∈I i∈I i∈I

Let a ∈ Ti∈I Ai and b ∈ S so that b ≤ a. Therefore, we have b ∈ Ti∈I Bi. Hence, i∈I Bi is a relative ordered (m, n)-hyperideal of S containing s.

Theorem 2.2. Suppose that (S,∘,≤) is an ordered semihypergroup and s ∈ A ⊆ S. Then, we have the following:

1. $[s]_A$ = (Si=1n si ∪sm ∘A∘sn$]_A$ for any positive integers m, n.
2. $[s]_A$ = (Si=1 si ∪sm ∘A$]_A$ for any positive integer m.
3. $[s]_A$ = (Si=1 si ∪sn$]_A$ for any positive integer n.

Proof

i) Let A ⊆ S. We have ($^{m+n}$ si ∪ s^m ∘ A ∘ s^n] = ∅. Suppose that a, b ∈ (i=1^n si ∪ s^m ∘ A ∘ s^n]A is such that a ≤ x and b ≤ y for some x, y ∈ i=1^n(s^i ∪ s^m ∘ A ∘ s$]_A$. If x, y ∈ s ∘ A ∘ s or x ∈ i=1 s, y ∈ s ∘ A ∘ s or x ∈ s ∘ A ∘ s, y ∈ i=1 s, then x∘y ⊆ s ∘S∘s. Therefore, x∘y ⊆ i=1 s ∪s ∘A∘s. It follows that a∘b ⊆ (i=1 s ∪s ∘A∘s$]_A$. Let x,y ∈ i=1 s. Then, x = s, y = s for some 1 ≤ p, q ≤

m+n. Now two cases arise: If $1 \leq p+q \leq m+n$, then $x \circ y \subseteq$ s. If $m + n < p + q$, then $x \circ y \subseteq s^m \circ A \circ A \circ s^n$. So, $x \circ y \subseteq (^{m+n} s^i \cup s^m \circ A \circ A \circ s^n]$. This implies that $(^{m+n} s^i \cup s^m \circ A \circ s^n]_A$ is a sub-semihypergroup of S. Moreover, we have the following:

$$
\begin{aligned}
(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^m \circ A &= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]_A \circ A \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \circ A \cup s^m \circ A \circ A]_A \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-1} \circ (s \circ A]_A \\
&= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-2} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]_A \circ (s \circ A]_A \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-2} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A \circ (s \circ A]_A \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-2} \circ (s^2 \circ A]_A \\
&\ \ \vdots \\
&\subseteq (s^m \circ A]_A.
\end{aligned}
$$

In a similar fashion, $A \circ (S^{m+n} s^i \cup s^m \circ A \circ s^n]^n \subseteq (A \circ s^n]$. Therefore, $(S^{m+n} s^i \cup s^m \circ A \circ s^n]^m \circ A \circ (^{m+n} s^i \cup s^m \circ A \circ s^n]^n \subseteq (s^m \circ A \circ s^n] \subseteq (^{m+n} s^i \cup s^m \circ A \circ s^n]$. So, $(^{m+n} s^i \cup s^m \circ A \circ s^n]$ is a relative (m, n)-hyperideal of S containing s. Hence $[s]m, n \subseteq (_{i=1}{}^n s^i \cup s^m \circ A \circ s^n]A$. For the reverse inclusion, suppose that $a \in$ (i=1n si$\cup$ s $\circ$ A $\circ$ s]A is such that $a \leq t$ for some $t \in$ (i=1 s $\cup$ s $\circ$ A $\circ$ s]A. If t = s for some $1 \leq j \leq m + n$, then $t \in [s]_{m,n}$ therefore, $a \in [s]_{m,n.}$ If $t \in s \circ A \circ s$, by the following relation:

$$s^m \circ A \circ s^n \subseteq ([s]A)^m \circ A \circ A \circ ([s]_A)^n \subseteq [s]_A,$$

then, we have $t \in [s]_{m,n.}$ Therefore, $a \in [s]_A$. This implies that $(S_{i=1}{}^n s^i \cup s^m \circ A \circ s^n] \subseteq [s]_A$. Hence, $[s]_{m,n} =$ (i=1 s $\cup$s $\circ$A$\circ$s$]_A$.
(ii) and (iii) can be proved in a similar fashion.

Lemma 2.3: Suppose that $(S,\circ,\leq)$ is an ordered semihypergroup, $A \subseteq S$ and $s \in A$. Further suppose that m, n are positive integers. Then, we have the following:
1. $([s]_{m,0})^m \circ A \subseteq (s^m \circ A]_A$.
2. $S \circ ([s]_{0,n})^n \subseteq (A \circ s^n]_A$.
3. $([s]_{m,n})^m \circ A \circ ([s]_{m,n})^n \subseteq (s^m \circ A \circ s^n]_A$.

Proof. (i) We have the following:

$$
\begin{aligned}
([s]_{m,0})^m \circ A &= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^m \circ A \\
&= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]_A \circ A \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \circ A \cup s^m \circ A \circ A]_A \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ A]^{m-1} \circ (s \circ A]_A \\
&\ \ \vdots \\
&\subseteq (s^m \circ A]_A.
\end{aligned}
$$

Hence, $([s]_A)^m \circ A \circ A \subseteq (s^m \circ A]_{A.}$ (ii) can be proved similarly as (i).
iii) We have the following:

Therefore, $([s]_{m,n})^m \circ A \subseteq (s^m \circ A]_{A.}$ In a similar fashion, $A \circ ([s]_{m,n})^n \subseteq (A \circ s^n]_{A.}$ Therefore, we have the following:

$$([s]_{m,n})^m \circ A \circ ([s]_{m,n})^n \subseteq (s^m \circ A]_A \circ ([s]_{m,n})^n \subseteq (s^m \circ (A \circ ([s]_{m,n})^n)]_A \subseteq (s^m \circ (A \circ s^n]_A]_A \subseteq (s^m \circ A \circ s^n]_A.$$

Hence, (iii) holds.

Theorem 2.4: Suppose that $(S,\circ,\leq)$ is an ordered semihypergroup, $A \subseteq S$ and m,n are positive integers. Let $R_{(m,0)}$ and $L_{(0,n)}$ be the set of all relative ordered (m,0)-hyperideals and the set of all relative ordered (0,n)-hyperideals of S, respectively. Then the following assertions are true:

i) S is relative (m,0)-regular if and only if for all $R \in R_{(m,0)}$, $R = (R^m \circ A]_A$.
ii) S is relative (0,n)-regular if and only if for all $L \in R_{(0,n)}$, $L = (A \circ L^n]_A$.

Proof.

i) Suppose that S is a relative (m,0)-regular. Then, we have

$$\forall s \in A, s \in (s^m \circ A]_A. \quad (1)$$

Suppose that $R \in R_{(m,0)}$. As $R^m \circ A \subseteq R$ and $R = (R]_A$, we have $(R^m \circ A]_A \subseteq R$. If $s \in R$, then by (1), we obtain $s \in (s^m \circ A]_A \subseteq (R^m \circ A]_A$. Therefore, $R \subseteq (R^m \circ A]_A$.

So, (Rm ∘A]A = R.

Conversely, suppose that

$$\forall R \in R_{(m,0)}, R = (R^m \circ A]_A. \quad (2)$$

Suppose that $s \in A$. Therefore, $[s]_{m,0} \in R_{(m,0)}$. Then by (2), we obtain the following:

$$[s]_{m,o} = (([s]_{m,0})^m \circ A]_A.$$

Applying Lemma 2.3, we obtain the following:

$$[s]_{m,o} \subseteq (s^m \circ A]_A,$$

Therefore, s ∈ (sm ∘A∘A]A. Hence, S is a relative (m,0)-regular.

(ii) It can be proved analogously.

Theorem 2.5: Suppose that $(S,\circ,\leq)$ is an ordered semihypergroup and m, n are non-negative integers. Let $A \subseteq S$. Suppose that $A_{(m,n)}$ is the set of all relative ordered (m, n)-hyperideals of S. Then, we have the following:

$$\text{S is (m,n)-regular} \Longleftrightarrow \forall A \in A_{(m,n)}, A = (A^m \circ A \circ A^n]_A \quad (3)$$

Proof: Consider the Following Four Cases:

Case (i): m = 0 and n = 0. Then, (3) implies

S is a relative (0,0)-regular $\Longleftrightarrow \forall A \in A_{(0,0)}, A = S$ because $A_{(0,0)} = \{S\}$ and S is relative (0, 0)-regular.

Case (ii): m = 0 and n = 0. Therefore, (3) implies

S is relative (0,n)-regular$\Longleftrightarrow \forall A \in A_{(0,n)}, A = (A \circ A^n]_A$. This follows by Theorem 2.4(ii).

Case (iii): m = 0 and n = 0. This can be proved applying Theorem 2.4(i).

Case (iv): m = 0 and n = 0. Suppose that S is relative (m,n)-regular. Therefore, we have the following:

$$\forall s \in A, s \in (sm \circ A \circ sn]_A. \quad (4)$$

Let $A \in A_{(m,n)}$. As $A^m \circ A \circ A^n \subseteq A$ and $A = (A]_A$, we obtain $(A^m \circ A \circ A^n]_A \subseteq A$. Suppose $s \in A$. Applying (4), $s \in (s^m \circ A \circ s^n]_A \subseteq (A^m \circ A \circ A^n]_A$. Therefore,

$A \subseteq (A^m \circ A \circ A^n]_A$. Hence $A = (A^m \circ A \circ A^n]_A$.

Conversely, suppose that $A = (A^m \circ A \circ A^n]_A$ for all $A \in A_{(m,n)}$. Suppose that $s \in A$. As $[s]_{m,n} \in A_{(m,n)}$, we have the following:

$$[s]_{m,n} = (([s]_{m,n})^m \circ A \circ ([s]_{m,n})^n]_A.$$

Applying Lemma 2.3(iii), we obtain $[s]_{m,n} \subseteq (s^m \circ A \circ s^n]_A$. Therefore, $s \in (s^m \circ A \circ s^n]_A$. Hence, S is a relative (m,n)-regular.

Theorem 2.6. Suppose that $(S,\circ,\leq)$ is an ordered semihypergroup and m and n are nonnegative integers. Suppose that $R_{(m,0)}$ and $L_{(0,n)}$ is the set of all relative (m,0)-hyperideals and relative (0,n)-hyperideals of S, respectively. Let $A \subseteq S$. Then, we have the following:

$$S \text{ is } (m,n)\text{-regular ordered semihypergroup} \Longleftrightarrow \forall R \in R_{(m,0)} \forall L \in L_{(0,n)}, \quad R \cap L = (R^m \circ L \cap R \circ L^n]_A. \qquad (5)$$

Proof. The Following are Four Cases:
Case (i): m = 0 and n = 0. Therefore, (5) implies
S is relative (0,0)-regular $\Longleftrightarrow \forall R \in R_{(0,0)} \forall L \in L_{(0,0)}, R \cap L = (L \cap R]_A$ because $R_{(0,0)} = L_{(0,0)} = \{S\}$ and S is relative (0,0)-regular.
Case (ii): m = 0 and n = 0. Therefore, (5) implies
S is relative (0,n)-regular $\Longleftrightarrow \forall R \in R_{(0,n)} \forall L \in L_{(0,n)}, R \cap L = (L \cap R \circ L^n]_A$. Suppose that S is relative (0,n)-regular. Suppose that $R \in R_{(0,0)}$ and $L \in L_{(0,n)}$. By Theorem 2.4(ii), $L = (A \circ L^n]_A$. As $R \in R_{(0,0)}$, we have R = S. Therefore $R \cap L = L$. Therefore,

$$(L \cap R \circ L^n]_A = (L \cap A \circ L^n]_A = ((A \circ L^n]_A \cap A \circ L^n]_A = (A \circ L^n]_A = L = R \cap L.$$

Conversely, suppose that

$$\forall R \in R_{(0,0)} \forall L \in L_{(0,n)}, R \cap L = (L \cap R \circ L^n]_A. \qquad (6)$$

If $R \in R_{(0,0)}$, then R = S. If $L \in L_{(0,n)}$, $A \circ L^n \subseteq L$ and $L = (L]_A$. Therefore, (6) implies that

$$\forall L \in L_{(0,n)}, L = (A \circ L^n]_A.$$

Applying Theorem 2.4(ii), S is relative (0,n)-regular.
Case (iii): m = 0 and n = 0. This can be proved as before.
Case (iv): m = 0 and n = 0. Suppose that S is relative (m,n)-regular. Suppose that $R \in R_{(m,0)}$ and $L \in L_{(0,n)}$. To prove that $R \cap L \subseteq (R^m \circ L]A \cap (R \circ L^n]_A$, suppose that $s \in R \cap L$. We now receive the following:

$$s \in (s^m \circ A \circ s^n]_A \subseteq (s^m \circ L]A \subseteq (R^m \circ L]_A \text{ and } s \in (s^m \circ A \circ s^n]_A \subseteq (R \circ s^n]_A \subseteq (R \circ L^n]_A.$$

Hence, $R \cap L \subseteq (R^m \circ L]_A \cap (R \circ L^n]_A$. As

$(R^m \circ L]_A \subseteq (R^m \circ A]_A \subseteq (R]_A = R$ and $(R \circ L^n]A \subseteq (A \circ L^n]_A \subseteq (L]_A = L$.

This implies that $(R^m \circ L]_A \cap (R \circ L^n]_A \subseteq R \cap L$. Therefore $R \cap L = (R^m \circ L]_A \cap (R \circ L^n]_A$. Conversely, suppose that

$$\forall R \in R_{(m,0)} \forall L \in L_{(0,n)}, R \cap L = (R^m \circ L \cap R \circ L^n]_A. \qquad (7)$$

Suppose that $R = [s]_{m,0}$ and L = S. Applying (7), we obtain the following: $[s]_{m,0} \subseteq (([s]_{m,0})^m \circ A]_A$. Applying Lemma 2.3, we obtain the following:

$$[s]_{m,0} \subseteq (s^m \circ A]_A. \qquad (8)$$

In a similar fashion, we obtain the following:

$$[s]_{0,n} \subseteq (A \circ s^n]_A. \qquad (9)$$

As $R^m \subseteq R$ and $L^n \subseteq L$, then by (7), we have the following:

$$\forall R \in R_{(m,0)} \forall L \in L_{(0,n)}, R \cap L \subseteq (R \circ L]_A.$$

As $(s^m \circ A]_A \in R_{(m,0)}$ and $(A \circ s^n]_A \in L_{(0,n)}$, we obtain the following:

$$(s^m \circ A]_A \cap (A \circ s^n]_A \subseteq ((s^m \circ A]_A \circ (A \circ s^n]A]_A \subseteq (s^m \circ A \circ s^n]_A.$$

By (8) and (9), we obtain the following:

$$[s]_{m,0} \cap [s]_{0,n} \subseteq (s^m \circ A \circ s^n]_A.$$

Hence, S is relative (m,n)-regular.

Conclusion
In this paper, relative ordered (m, n)-hyperideals in ordered semihypergroups has been studied. Then, relative ordered (m, 0)-hyperideals and relative ordered (0, n)-hyperideals as well as characterization of regular ordered semihypergroups has been studied. We then obtained some results based on these relative hyperideals. We showed that the intersection of all relative ordered (m, n)-hyperideals of S containing s is a relative ordered (m, n)-hyperideal of S containing s. It was shown that (S,∘,≤) is an ordered semihypergroup, where A ⊆ S and m,n are positive integers. We showed that if R(m,0) and L(0,n) be the set of all relative

ordered (m,0)-hyperideals and the set of all relative ordered (0,n)-hyperideals of S, respectively. Then the following assertions are true: i) S is relative (m,0)-regular if and only if for all R ∈ R(m,0), R = (Rm ◦A]A. ii) S is relative (0,n)-regular if and only if for all L ∈ R(0,n), L = (A◦Ln]A. Furthermore, it was shown that if (S, ◦, ≤) is an ordered semihypergroup and m, n are non-negative integers. Let A ⊆ S, A(m,n) is the set of all relative ordered (m, n)-hyperideals of S. Then: S is (m, n)-regular ⇐⇒ ∀A ∈ A(m,n),A = (Am ◦A◦An]A.

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