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An Optimum Solution of Matrix Game with GTIFN by Solving Dominance Property

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Abstract
This Paper Proposes two methods to solve Transportation Problem in which all the cost coefficients demands and supplies are taken as Trapezoidal Fuzzy Number and Trapezoidal Intuitionistic Fuzzy Number, without converting to classical transportation problem. We yield the initial basic feasible solution and optimal solution by conventional optimization process. The numerical example illustrates the efficiency of the proposed technique. The transportation problem is one of the earliest applications of linear programming problems. Transportation models have wide applications in logistic and supply chain for reducing the cost efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. In this paper proposes the method to solve dominance property consider the matrix game taken as generalized trapezoidal intuitionistic fuzzy number are ranking technique. This matrix game solved by dominance property to find the value of the game.

Keywords: Intuitionistic fuzzy set, trapezoidal fuzzy number, transportation problem, value of the game

Introduction

In today's real life, they are many complex situations in engineering and business, in which experts and decision markers struggle with uncertainty and hesitation. In many practical situations, collection of crisp data of various parameters is difficult due to lack of exact communications, error in data, market knowledge and customer's satisfaction. The information available is sometimes vague and insufficient. The real life problems, when defined by the decision marker with uncertainty leads to the notion of fuzzy sets. Due to imprecise information the exact evaluation of membership values is not possible. Moreover, the evaluation of the non-membership values is always impossible. This leads to an indeterministic environment where hesitation survives, dealing with inexact information while making decisions, the concept of fuzziness was introduced by Bellman and Zadeh [1]. K.T, Atanassov [2] introduced the concept of intuitionistic fuzzy set theory, which is more apt to deal with such problems. Chetia. K and P.K. Das [3] proved some results on intuitionistic fuzzy soft matrix. Intuitionistic fuzzy sets [4, 5, 6] found to be highly effective in dealing with vengeances, among several higher order fuzzy sets. This paper formulates the dominance property generalized trapezoidal intuitionistic fuzzy number as costs to deal with uncertainty and hesitation in supply and demand. The new ranking measure proposed in this paper proves to be efficient over the other fuzzy ranking existing techniques.

Preliminaries

Fuzzy Set

If X is a universe of discourse and $\mu_{\bar{A}}$ is a membership function from X to the unit interval [0, 1] then the fuzzy set $\bar{A}$ defined as

$$\bar{A} = \{(x, \mu_{\bar{A}}(x) : x \in X\}$$

The membership function $\mu_{\bar{A}}(x)$ is called the membership value (grad or degree) of x∈X in the fuzzy set $\bar{A}$. These membership grad are often represented by real numbers belonging to the closed interval [0, 1].

Intuitionistic Fuzzy Set

Let X be the universal set. An intuitionistic fuzzy set (IFS) $\tilde{A}^I$ in X is given by $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x) : x \in X\}$ where the functions $\mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)$ define respectively, the degree of membership and degree of non-membership of the element x∈X to the set $\tilde{A}^I$, which is a subset of X and for every x∈X, $0 \le \mu_{\tilde{A}^I}(x) + \upsilon_{\tilde{A}^I}(x) \le 1$. Obviously, every intuitionistic fuzzy set has the form $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)) : x \in X\}$ in X. for each intuitionistic fuzzy set $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)) : x \in X\}$, $\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \upsilon_{\tilde{A}^I}(x)$ is called the hesitancy degree of x to lie in. If $\tilde{A}^I$ is a fuzzy set, then

$$\pi_{\tilde{A}^I}(x) = 0 \; for \; all \; x \in X$$

Generalized Trapezoidal Intuitionistic Fuzzy Number

An intuitionistic fuzzy number $\tilde{A}^I$ is said to be generalized trapezoidal fuzzy number (GTIFN) with parameters $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and denoted by

$$\tilde{A}^I = (b_1, a_1, b_2, a_2, b_3, a_3, a_4, b_4; \omega_{\tilde{A}^I}, \upsilon_{\tilde{A}^I})$$
$$or$$
$$\tilde{A}^I = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_{\tilde{A}^I}, \upsilon_{\tilde{A}^I})$$

If its membership and non-membership functions are as follows

$$\mu_{\tilde{A}^I} = \begin{cases} 0, & if\ x < a_1 \\ \omega_{\tilde{A}^I}\left(\frac{x-a_1}{a_2-a_1}\right), & if\ a_1 \le x \le a_2 \\ \omega_{\tilde{A}^I}, & if\ a_2 \le x \le a_3 \\ \omega_{\tilde{A}^I}\left(\frac{a_4-x}{a_4-a_3}\right), & if\ a_3 \le x \le a_4 \\ 0, & if\ x > a_4 \end{cases}$$

$$\upsilon_{\tilde{A}^I} = \begin{cases} 1, & if\ x < b_1 \\ \left(\frac{(b_2-x)+u_{\tilde{A}^I}(x-b_1)}{b_2-b_1}\right), & if\ b_1 \le x \le b_2 \\ \upsilon_{\tilde{A}^I}, & if\ b_2 \le x \le b_3 \\ \left(\frac{(x-b_3)+u_{\tilde{A}^I}(b_4-x)}{b_4-b_3}\right), & if\ b_3 \le x \le b_4 \\ 1, & if\ x > b_4 \end{cases}$$

Where $0<\omega_{\tilde{A}^I} \le 1, 0 \le \upsilon_{\tilde{A}^I} \le 1\ and\ 0 < \omega_{\tilde{A}^I} + \upsilon_{\tilde{A}^I} \le 1.$

Ranking Technique

Let $\tilde{A}^I = ((a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2): \omega_{\tilde{A}^I}, \upsilon_{\tilde{A}^I})$ be a generalized trapezoidal intuitionistic fuzzy number, then we define the new ranking of $\tilde{A}^I$ as,

$R_1(\tilde{A}^I) = \frac{\omega_{\tilde{A}^I}(a_1+b_1+c_1+d_1)}{4}$ and $R_2(\tilde{A}^I) = \frac{\upsilon_{\tilde{A}^I}(a_2+b_2+c_2+d_2)}{4}$

and $R(\tilde{A}^I) = \frac{R_1(\tilde{A}^I)+2R_2(\tilde{A}^I)}{3}$

Dominance Property

Sometimes it is observed that one of the pure strategies of either player is always inferior to at least one of the remaining ones the superior strategies are said to dominate the forier ones.

General Rules

i) If all the demand of a row, say k[th] are less than (or) equal to the corresponding elements of any other row say r[th] then k[th] row is dominated by r[th] row,
ii) If all the element of a column say k[th] are greater than (or) equal to the corresponding elements of any other column say r[th] then k[th] column is dominated by the r[th] column.
iii) Dominated rows (or) column may be deleted to reduce the size of pay off matrix as the optimal strategies will remain unaffected

The Modified Dominance Property

The dominance property is not always based on the superiority of pure strategies only. A given strategy also be said to be dominated if it is infusion of 2 (or) more other pure strategies.

More generally if some convex linear combination of some rows dominates the i[th]row then i[th] row will be deleted similar arguments follows for columns.

Numerical Example

Consider a fuzzy transportation problem with four origins O_1, O_2, O_3, O_4 and four destinations D_1, D_2, D_3, D_4 whose costs are considered to be generalized trapezoidal intuitionistic fuzzy numbers.

Table 1: Table showing ranking technique graphical method value of the game.

	D_1	D_2	D_3	D_4	Supply
O_1	(3,5,6,8; 2,4,7,10; 0.6,0.1)	(5,8,11,13; 4,6,12,14; 0.7,0.2)	(8,10,11,15; 7,9,13,17; 0.5,0.3)	(5,8,10,12; 4,7,11,13; 0.5,0.3)	(5,8,10,12; 4,7,11,14; 0.5,0.3)
O_2	(2,4,5,7; 1,3,6,8; 0.6,0.1)	(3,5,6,8; 1,4,7,10; 0.4,0.3)	(6,8,10,12; 5,7,11,13; 0.7,0.1)	(5,8,10,12; 4,6,11,13; 0.8,0.1)	(5,8,9,11; 4,6,10,12; 0.4,0.3)
O_3	(7,9,10,12; 6,8,11,123; 0.7,0.1)	(5,7,10,12; 4,6,11,14; 0.7,0.1)	(8,11,13,15; 7,9,14,1; 0.6,0.2)	(9,8,7,10; 6,7,8,11; 0.8,0.1)	(5,7,9,11; 4,6,10,12; 0.6,0.2)
O_4	(5,7,8,10; 4,6,9,11; 0.6,0.1)	(2,5,6,8; 1,3,7,9; 0.7,0.1)	(5,7,10,14; 4,6,12,15; 0.6,0.2)	(2,4,5,7; 1,3,6,8; 0.7,0.1)	(4,6,8,10; 3,5,9,11; 0.6,0.2)
Demand	(6,8,10,12; 5,7,11,13; 0.8,0.1)	(5,7,8,10; 4,6,9,11; 0.6,0.1)	(10,8,6,4; 12,9,5,3; 0.4,0.3)	(7,9,11,14; 6,8,13,15; 0.5,0.3)	(4,7,8,10; 3,5,9,11; 0.5,0.3)

Solution

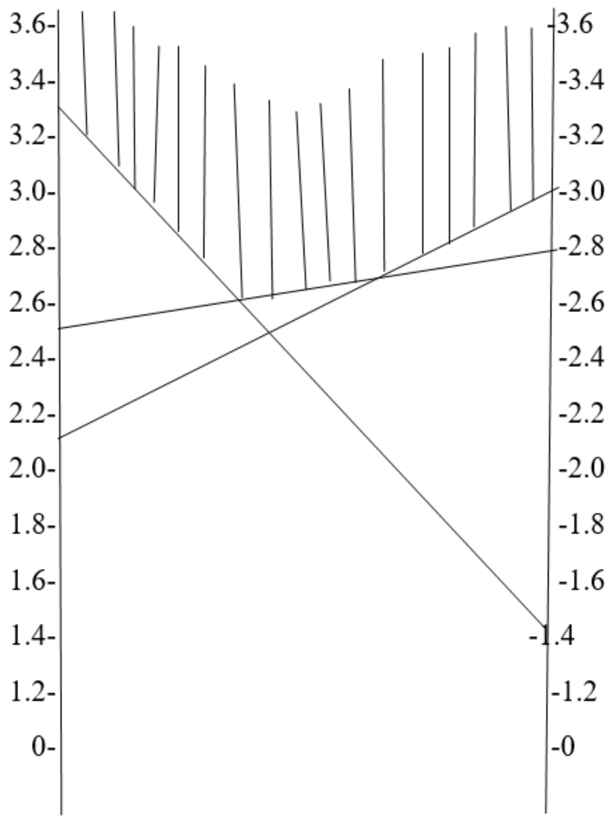
Using ranking technique the rank of generalized trapezoidal intuitionistic fuzzy cost matrix is obtained.

$$\begin{pmatrix} 1.4833 & 3.3583 & 4.1333 & 3.2083 & 3.2583 \\ 1.2 & 1.8333 & 2.7 & 2.9 & 2.7 \\ 2.85 & 2.566 & 3.8833 & 2.8 & 2.6667 \\ 2.1 & 1.5583 & 2.0333 & 1.35 & 2.3333 \\ 3 & 2.1 & 3.8083 & 2.6083 & 2.6667 \end{pmatrix}$$

Proceeding by dominance property, the original 5×5 matrix can be reduced to 3×2 matrix, i.e., the payoff matrix.

$$\begin{bmatrix} 1.4833 & 3.3583 \\ 2.85 & 2.5666 \\ 3 & 2.1 \end{bmatrix}$$

The given reduced matrix does not possess any saddle point so by using graphical method



The cost matrix 3×2 can be reduced by 2×2 matrix, then find the value of the game,

$$\begin{bmatrix} 2.85 & 2.5666 \\ 3 & 2.1 \end{bmatrix}$$

By using mixed strategy formula, we get
The value of the game $\gamma = 2.78$

Conclusion

This paper proposes a value of the game. Whose costs are taken as generalized trapezoidal intuitionistic fuzzy numbers. For future research we proposes effective implementation of the trapezoidal fuzzy numbers in all fuzzy problems. The same approach of solving the intuitionistic fuzzy problems may also be utilized in future studied of operational research.

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