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A Critical Review Report on Cosmological Inflation, Modified Cosmology and Energy Conditions in $f(Q, L_m)$ Gravity

*[1]Kajal Parate and [2]GU Khapekar

*[1]Department of Science & Technology, GH Raisoni Skill Tech University, Nagpur, Maharashtra, India.

[2]Department of Mathematics, Jagadamba Mahavidyalaya, Achalpur City, Amravati, Maharashtra, India.

Abstract

The discovery of the accelerated expansion of the universe has motivated the development of several modified theories of gravity that extend General Relativity (GR). Among these theories, $f(Q, Lm)$ gravity has emerged as a promising framework by introducing a non-minimal coupling between the non-metricity scalar Q and the matter Lagrangian L_m. Unlike curvature-based or torsion-based formulations, this theory is constructed within the symmetric teleparallel geometry, where gravitation is entirely described by non-metricity. Recent studies have demonstrated that this framework provides successful descriptions of both the inflationary epoch of the early universe and the present accelerated cosmic expansion while satisfying important theoretical and observational constraints. Furthermore, the formulation modifies the classical energy conditions, providing new insights into the nature of dark energy and the evolution of the evaluate the strengths, limitations, observational compatibility, and future prospects of the proposed models. The review highlights that $f(Q, Lm)$ gravity represents a viable alternative to General Relativity and offers a unified framework capable of explaining both early and late-time cosmic acceleration.

Keywords: Modified Gravity, $f(Q, Lm)$ Gravity, Cosmological Inflation, Non-Metricity, Energy Conditions, Symmetric Teleparallel Gravity, Dark Energy, Cosmic Acceleration.

1. Introduction

Modern cosmology has experienced remarkable progress over the last few decades due to increasingly precise astronomical observations. Measurements of the cosmic microwave background radiation (CMB), Type Ia supernovae, baryon acoustic oscillations (BAO), and large-scale structure surveys have established that the universe has undergone two distinct periods of accelerated expansion. The first occurred during the earliest moments of cosmic evolution as cosmological inflation [5-9], whereas the second corresponds to the present epoch of late-time cosmic acceleration [10-17]. Although the standard cosmological model based on General Relativity (GR) [1] and the cosmological constant (ΛCDM model) successfully explains many observational phenomena, it continues to face several theoretical challenges, including the cosmological constant problem, fine-tuning problem, coincidence problem, and the absence of a complete explanation for dark energy.

To overcome these limitations, several modified theories of gravity have been proposed in recent years. These theories extend Einstein's gravitational action by replacing the Ricci scalar with more general functions involving curvature, torsion, or non-metricity. Among the recently developed approaches, symmetric teleparallel gravity [20, 21] has attracted considerable attention because gravitation is entirely described through the non-metricity scalar Q[20, 21], while both curvature and torsion vanish. A further extension of this framework is the $f(Q, L_m)$ gravity theory, where the gravitational action depends simultaneously on the non-metricity scalar and the matter Lagrangian. This non-minimal coupling between geometry and matter modifies the gravitational interaction and naturally produces deviations from General Relativity that may explain both early- and late-time cosmic acceleration [10-17].

Motivated by the aforementioned studies of [32, 33, 34], we review three different cosmological aspects of $f(Q, L_m)$ gravity. Section 2 thoroughly examines inflationary dynamics in the early universe. We develop a modified cosmological model to explain late-time acceleration using observational datasets. Also, we examine the validity of classical energy conditions within this modified gravitational framework. Together, these studies provide a comprehensive understanding of the theoretical structure and cosmological applications of $f(Q, L_m)$ gravity. Section 3, we explore the critical analysis of cosmological inflation, modified cosmology and energy conditions. Finally, Section 4, provides a summary and

*Corresponding Author: Kajal Parate

concluding observations.

2. $f(Q, L_m)$ Gravity

2.1. Cosmological Inflation:

Cosmological inflation has become an essential component of the modern cosmological paradigm because it successfully resolves several shortcomings of the standard Big Bang model, including the horizon, flatness, and monopole problems. Inflation also provides the physical mechanism responsible for generating the primordial density fluctuations that eventually formed galaxies and large-scale structures observed today. In conventional inflationary models, this accelerated expansion is driven by a slowly evolving scalar field known as the inflation. The dynamics of this field are governed by the slow-roll approximation, which predicts observable quantities such as the scalar spectral index (n_s) and the tensor-to-scalar ratio (r). Although General Relativity successfully describes gravitational interactions over a wide range of physical scales, it does not provide a complete theoretical explanation for the origin of inflation or the dark sector of the universe. Consequently, modified gravity theories have been extensively explored as possible alternatives capable of addressing these limitations.

The reviewed inflationary study investigates cosmological inflation [5-9] within the framework of $f(Q, L_m)$ gravity by considering both linear and nonlinear matter-geometry coupling models. Unlike conventional inflationary theories, the coupling between the non-metricity scalar [20, 21] and the matter Lagrangian modifies the Friedmann equations and the Klein–Gordon equation governing the evolution of the inflation field. These modifications alter the slow-roll dynamics and consequently influence the inflationary observables.

The study derives analytical expressions for the slow-roll parameters, scalar spectral index, tensor-to-scalar ratio, and the number of e-foldings using a power-law inflation potential. Comparison with the latest Planck [12], BK15, and BAO observational datasets demonstrates that both models can reproduce observationally acceptable inflationary predictions under suitable parameter choices. The linear model produces viable inflationary solutions for positive coupling parameters, whereas the nonlinear model remains compatible with observations primarily for negative parameter values. These results indicate that $f(Q, L_m)$ gravity provides a competitive alternative framework for describing the inflationary universe while remaining consistent with current cosmological observations. Shiravand *et al.* [32] investigated cosmological inflation in $f(Q, L_m)$ gravity and showed that the model is consistent with recent observational data.

2.2. Modified Cosmology

The observation that the universe is presently undergoing accelerated expansion has become one of the most significant discoveries in modern astrophysics. Within the standard ΛCDM model, this acceleration is attributed to dark energy in the form of the cosmological constant. However, despite its observational success, the cosmological constant suffers from several unresolved theoretical issues, including the fine-tuning and coincidence problems. These difficulties have motivated researchers to investigate modified theories of gravity capable of explaining cosmic acceleration without introducing an additional dark energy component.

Among these alternatives, $f(Q, L_m)$gravity extends symmetric teleparallel gravity [20, 21] by introducing an explicit interaction between non-metricity and matter. The reviewed study considers a nonlinear cosmological model characterized by a power-law dependence on the matter Lagrangian. The modified Friedmann equations are derived for a spatially flat Friedmann–Lemaitre–Robertson–Walker (FLRW) universe filled with pressure less matter.

Analytical solutions for the Hubble [14] parameter, effective energy density, effective pressure, and deceleration parameter are obtained, allowing the cosmological evolution to be investigated throughout different epochs. To examine the observational viability of the model, the authors constrain the free parameters using Hubble observations, Pantheon+ [13] Type Ia supernova data, and their combined datasets.

The statistical analysis demonstrates that the model successfully reproduces the observed expansion history of the universe and predicts the transition from cosmic deceleration to acceleration at a redshift consistent with observational measurements. Furthermore, deviations of the coupling parameter from its General Relativity value indicate the presence of measurable modifications to gravitational dynamics. These results suggest that $f(Q, L_m)$ gravity offers a promising explanation for late-time cosmic acceleration [10-17] while remaining compatible with current astronomical observations. Myrzakulov *et al.* [33] proposed a modified cosmological model in $f(Q, L_m)$ gravity to explain the late-time accelerated expansion of the Universe.

2.3. Energy Conditions

Energy conditions constitute one of the fundamental theoretical tools in gravitational physics because they determine whether matter behaves in a physically reasonable manner and whether gravity remains attractive under different spacetime conditions. Within General Relativity, the weak, null, dominant, and strong energy conditions [2-4, 25, 34] play essential roles in black hole thermodynamics, singularity theorems, and cosmological evolution. However, modified gravitational theories alter the effective energy density and pressure appearing in the field equations, making it necessary to re-examine these conditions.

The reviewed study investigates the classical energy conditions within the framework of $f(Q, L_m)$gravity by analyzing two representative models involving linear and nonlinear matter-geometry couplings. Using the generalized Friedmann equations together with the Raychaudhuri equation formulated in Weyl geometry, the study derives modified expressions for the effective energy density and pressure that govern the evolution of the universe.

The numerical analysis demonstrates that the weak, null, and dominant energy conditions remain satisfied over wide regions of parameter space, whereas the strong energy condition is naturally violated. This violation is particularly significant because it provides the theoretical requirement necessary to explain the observed accelerated expansion of the universe. Moreover, the equation-of-state parameter exhibits transitions between quintessence, cosmological constant, and phantom phases depending upon the coupling parameters, illustrating the flexibility of the theory in describing different dark energy scenarios.

Overall, the investigation establishes that $f(Q, L_m)$ gravity successfully accommodates the necessary violations of the strong energy condition while preserving the physical consistency of the remaining energy conditions. Consequently, the theory represents a viable modified gravity model capable of explaining the late-time evolution of the universe without conflicting with fundamental physical principles. Myrzakulov *et al.* [34] studied the classical energy conditions in $f(Q, L_m)$ gravity and demonstrated the validity of WEC, NEC, and DEC, while the SEC is naturally violated.

3. Critical Analysis

The three studies reviewed in this paper investigate different aspects of $f(Q, L_m)$ gravity while sharing a common objective of exploring the cosmological implications of the non-minimal coupling between non-metricity and matter. Although each study focuses on a different physical problem—namely cosmological inflation [5-9], late-time cosmic evolution, and energy conditions — they collectively demonstrate the capability of $f(Q, L_m)$ gravity to provide an alternative description of gravitational phenomena beyond General Relativity (GR) [1]. This section critically evaluates the methodology, theoretical framework, observational consistency, strengths, and limitations of each work.

3.1. Critical Analysis of Cosmological Inflation in $f(Q, L_m)$ Gravity

The study on cosmological inflation [5-9, 32] extends the application of $f(Q, L_m)$ gravity to the earliest stage of the universe. Inflation has become an indispensable component of modern cosmology because it successfully explains several shortcomings of the standard Big Bang model, including the horizon problem, flatness problem, and monopole problem. Rather than introducing an entirely new inflationary mechanism, the authors incorporate the inflationary scalar field into the modified gravitational framework based on symmetric teleparallel geometry.

One of the major strengths of this work is the formulation of inflation within a theory where gravity is governed by the non-metricity scalar [20, 21] instead of spacetime curvature. This geometric interpretation provides a fresh perspective on gravitational interactions while maintaining mathematical consistency with symmetric teleparallel gravity [20, 21].

The authors investigate two different functional forms of

$$f(Q, L_m): f(Q, L_m) = -\alpha Q + 2L_m + \beta$$

and

$$f(Q, L_m) = -\alpha Q + (2L_m)^2 + \beta$$

studying both linear and nonlinear matter couplings enables a direct comparison between minimal and strong matter-geometry interactions. This comparative approach increases the reliability of the conclusions because it demonstrates how inflationary predictions change with the coupling structure. Another significant contribution of the study is the derivation of analytical expressions for the slow-roll parameters, scalar spectral index (n_s), tensor-to-scalar ratio (r), and e-folding number. These quantities are among the most important observational parameters in inflationary cosmology because they can be directly compared with measurements of the cosmic microwave background radiation. Unlike many theoretical inflationary models that remain purely mathematical, this work validates its predictions using Planck [12], BK15, and BAO observational datasets. The comparison demonstrates that the linear model successfully reproduces the observed inflationary parameters when positive values of the coupling constants are considered. The nonlinear model also agrees with observations but only within a restricted parameter space characterized by negative coupling parameters. The study therefore establishes that $f(Q, L_m)$gravity possesses sufficient flexibility to generate realistic inflationary scenarios while remaining compatible with present cosmological observations. Despite these strengths, several limitations remain.

The analysis assumes a power-law scalar potential throughout the investigation. Although power-law potentials are widely used in inflationary cosmology because of their mathematical simplicity, other inflationary potentials such as natural inflation, Higgs inflation, Starobin sky inflation, plateau models, and exponential potentials are not investigated. Consequently, the general applicability of the proposed theory remains uncertain. Another limitation is that the analysis is entirely restricted to the slow-roll approximation. Although this approximation accurately describes most inflationary models, it neglects the reheating stage following inflation and the possible deviations from slow-roll evolution that may influence primordial perturbations. Furthermore, the study primarily focuses on background cosmological evolution. A more detailed investigation of scalar perturbations, non-Gaussianity, reheating mechanisms, and quantum corrections would provide a more comprehensive understanding of inflation within $f(Q, L_m)$ gravity. Nevertheless, this work successfully demonstrates that modified non-metricity gravity can naturally explain the inflationary epoch while remaining consistent with high-precision cosmological observations.

3.2. Critical Analysis of Modified Cosmology in $f(Q, L_m)$ Gravity

The second study investigates the late-time cosmological evolution of the universe within the framework of $f(Q, L_m)$ gravity [33]. Unlike the inflationary paper, which concentrates on the early universe, this work addresses one of the most important challenges in modern cosmology—the accelerated expansion of the present universe. A major advantage of the proposed cosmological model is its ability to explain cosmic acceleration without explicitly introducing dark energy as an independent component. Instead, the accelerated expansion emerges naturally from the interaction between the non-metricity scalar [20, 21] and the matter Lagrangian.

The nonlinear model $f(Q, L_m) = -Q + \alpha L_m^n + \beta$ contains three independent parameters that allow sufficient flexibility to reproduce different cosmological histories. In particular, the exponent n controls the strength of the matter-geometry interaction and determines the deviation from General Relativity. An important contribution of this work is the derivation of exact analytical solutions for the modified Friedmann equations. Analytical solutions are generally preferred over purely numerical calculations because they reveal the physical dependence of cosmological evolution on the model parameters.

Another strength of the study lies in its observational analysis. The authors constrain the free parameters using three independent observational datasets:

- Hubble [14] parameter observations,
- Pantheon+ [13] Supernova dataset,
- Combined Hubble [14] and Pantheon+ [13] observations.

The inclusion of multiple datasets significantly improves the reliability of the statistical analysis because the best-fit parameters are constrained using complementary cosmological observations. The predicted transition redshift $z_t \approx 0.6 - 0.8$ is in excellent agreement with independent observational studies. Similarly, the present-day value of the deceleration parameter confirms that the universe is currently undergoing accelerated expansion. One particularly interesting outcome of the analysis is that the best-fit values satisfy $n \neq 1$ indicating measurable deviations from Einstein's General Relativity. This result supports the hypothesis that modified gravity may contribute to explaining the observed expansion history of the universe.

The study also discusses the Hubble [14] tension, one of the most important unresolved problems in cosmology. Since different values of the present Hubble constant can be accommodated within the model, $f(Q, L_m)$ gravity may provide an alternative explanation for this discrepancy. Despite these promising results, the model also possesses several limitations.

Only pressure less matter is considered throughout the analysis. The effects of radiation, neutrinos, and relativistic matter are neglected, restricting the applicability of the model to the late universe. Similarly, the analysis assumes spatial flatness throughout cosmic evolution. Although current observations strongly support a nearly flat universe, extending the theory to non-flat geometries would improve its generality. The model parameters are constrained using only background cosmological observations. Future studies should incorporate additional observational probes such as weak gravitational lensing, cosmic shear, redshift-space distortions, galaxy clustering, and gravitational-wave standard sirens. Overall, this study successfully demonstrates that $f(Q, L_m)$ gravity represents a promising alternative to the standard ΛCDM cosmology while simultaneously providing a possible explanation for late-time acceleration and the Hubble [14] tension.

3.3. Critical Analysis of Energy Conditions in $f(Q, L_m)$ Gravity

The third study investigates one of the most fundamental theoretical aspects of modified gravity—the validity of energy conditions [2-4, 25, 34]. Energy conditions are essential because they determine whether matter behaves physically and whether gravity remains attractive. They also play an important role in singularity theorems, black-hole thermodynamics, and gravitational collapse. Unlike many previous investigations that focus solely on cosmological evolution, this work emphasizes the theoretical consistency of $f(Q, L_m)$ gravity. The authors investigate four classical energy conditions:

- Weak Energy Condition (WEC),
- Null Energy Condition (NEC),
- Dominant Energy Condition (DEC),
- Strong Energy Condition (SEC).

The derivation is based on the generalized Raychaudhuri equation within Weyl geometry, making the mathematical formulation considerably more rigorous than standard analyses based solely on Friedmann equations. A major strength of this work is its simultaneous investigation of two different $f(Q, L_m)$ models. Comparing linear and nonlinear couplings enables the authors to evaluate how the strength of matter-geometry interaction influences the physical viability of the theory. The numerical analysis reveals that the WEC, NEC, and DEC remain satisfied over large regions of parameter space, while the SEC is violated. This result is particularly important because accelerated cosmic expansion requires violation of the strong energy condition. Consequently, the theory naturally produces cosmic acceleration without violating the remaining physically acceptable energy conditions. The study also investigates the evolution of the effective equation-of-state parameter. Interestingly, the nonlinear model exhibits transition between the following,

- Quintessence,
- Cosmological Constant,
- Phantom regimes.

These transitions indicate that $f(Q, L_m)$ gravity possesses sufficient dynamical richness to describe multiple phases of dark-energy evolution. Another important contribution is the comparison between the linear and nonlinear models.

The linear model is mathematically simpler and easier to interpret, whereas the nonlinear model provides greater flexibility and stronger violations of the strong energy condition. Consequently, the nonlinear model appears more suitable for explaining the observed accelerated expansion of the universe. Despite these contributions, the study also has several limitations.

The analysis remains primarily theoretical and does not compare the energy-condition constraints with observational cosmological data. Similarly, only homogeneous and isotropic cosmological models are considered. Extensions to anisotropic cosmologies, black-hole spacetimes, wormholes, neutron stars, and compact objects could further test the physical consistency of $f(Q, L_m)$ gravity. Finally, the physical origin of the matter-geometry coupling remains phenomenological. A deeper derivation from fundamental quantum gravity or field theory would strengthen the theoretical foundation of the model.

Nevertheless, this work provides an important theoretical validation of $f(Q, L_m)$ gravity by demonstrating that the theory remains physically consistent while naturally producing accelerated expansion.

3.4. Comparative Discussion

Although the three investigations address different cosmological problems, they are closely connected through the common framework of $f(Q, L_m)$ gravity.

The inflationary study focuses on the early universe, where the matter-geometry coupling modifies the dynamics of the inflation field and generates observationally viable inflation. The modified cosmology study concentrates on the late universe, showing that the same theoretical framework successfully reproduces the observed accelerated expansion without requiring an explicit cosmological constant. The energy-condition analysis provides the theoretical foundation explaining why accelerated expansion naturally arises within the modified gravity framework. The violation of the strong energy condition offers a consistent physical mechanism for cosmic acceleration while preserving the remaining classical energy conditions.

Collectively, these three studies demonstrate that $f(Q, L_m)$ gravity possesses the potential to describe both the earliest and latest stages of cosmic evolution within a single theoretical framework. The theory modifies the gravitational interaction through non-metricity rather than curvature, introduces non-minimal matter-geometry coupling, and remains compatible with current cosmological observations. However, several challenges remain before the theory can be considered a complete alternative to General Relativity. These include a deeper understanding of the physical origin of the coupling function, investigations of cosmological perturbations, extensions to astrophysical systems such as black holes and neutron stars, and further observational constraints using future cosmological surveys.

Table 1: Comparative Analysis of the Three Reviewed Papers

Feature	Cosmological Inflation	Modified Cosmology	Energy Conditions
Primary Objective	Early Universe Inflation	Late-time Cosmic Expansion	Physical Consistency of Gravity
Gravity Framework	$f(Q, L_m)$	$f(Q, L_m)$	$f(Q, L_m)$
Geometry	Symmetric Teleparallel	Symmetric Teleparallel	Weyl–Symmetric Teleparallel
Main Equations	Slow-roll & Modified Friedmann	Modified Friedmann	Raychaudhuri & Modified Friedmann
Observational Analysis	Planck + BK15 + BAO	Hubble + Pantheon+	Mainly theoretical
Major Strength	Inflation compatible with observations	Explains cosmic acceleration	Validates physical consistency
Limitation	Limited inflation potential	Only late-time cosmology	Limited observational testing
Future Scope	Reheating, perturbations	Structure formation, GW constraints	Black holes, wormholes, compact stars

4. Summary and Conclusion

The rapid development of observational cosmology has significantly enhanced our understanding of the universe while simultaneously exposing several theoretical limitations of General Relativity (GR) [1]. Although the standard ΛCDM cosmological model remains highly successful in explaining a wide range of observations, unresolved issues such as the horizon problem, flatness problem, dark energy, and the Hubble [14] tension continue to motivate the search for alternative theories of gravity. Among these alternatives, $f(Q, L_m)$ gravity has emerged [20–22, 27–31] as a promising framework by extending symmetric teleparallel gravity [20, 21] through a non-minimal coupling between the non-metricity scalar Q [20, 21] and the matter Lagrangian L_m. This coupling modifies the gravitational interaction and provides a unified approach for investigating both the early and late stages of cosmic evolution.

This review has presented a comprehensive discussion of three recent studies that explore different cosmological applications of $f(Q, L_m)$ gravity. Collectively, these investigations demonstrate that the theory possesses sufficient mathematical flexibility and physical consistency to explain several important cosmological phenomena without relying exclusively on General Relativity.

The first study examined cosmological inflation [5-9, 32] within the framework of $f(Q, L_m)$ gravity. By introducing linear and nonlinear matter-geometry coupling models, the authors derived modified slow-roll parameters, scalar spectral indices, tensor-to-scalar ratios, and e-folding numbers. The predicted inflationary observables were compared with the latest Planck [12], BK15, and BAO datasets, showing that both models can satisfy observational constraints under suitable choices of the coupling parameters. These results indicate that $f(Q, L_m)$ gravity provides an effective description of the inflationary universe while remaining compatible with current cosmological observations.

The second study focused on late-time cosmological evolution [23, 24, 26, 33]. The modified Friedmann equations derived within the $f(Q, L_m)$ framework successfully reproduce the observed accelerated expansion of the universe without requiring an explicit cosmological constant. Statistical analyses using Hubble [14] parameter measurements and the Pantheon+ [13] supernova dataset demonstrated good agreement between theoretical predictions and observational data. The model also predicts a transition from cosmic deceleration to acceleration at a redshift consistent with current observations and offers a possible contribution toward resolving the Hubble tension.

The third study investigated the classical energy conditions [2-5, 25, 34] within the same theoretical framework. By deriving generalized expressions for the effective energy density and pressure, the authors demonstrated that the weak, null, and dominant energy conditions remain satisfied over broad regions of parameter space, while the strong energy condition is naturally violated. This violation provides a physically consistent explanation for the observed accelerated expansion of the universe and supports the role of matter-geometry coupling as an alternative mechanism to dark energy.

A comparative evaluation of these three investigations reveals several common conclusions. First, the introduction of non-minimal coupling between geometry and matter produces significant modifications to gravitational dynamics while preserving the overall consistency of the cosmological model. Second, the symmetric teleparallel formulation offers an alternative geometric description of gravity in which non-metricity replaces curvature as the fundamental gravitational quantity. Third, the theoretical predictions remain broadly compatible with current observational constraints, indicating that $f(Q, L_m)$ gravity represents a viable extension of General Relativity.

Despite these promising results, several open questions remain. Most existing analyses focus on homogeneous and isotropic cosmological backgrounds, whereas more realistic cosmological scenarios require investigations of perturbation theory, structure formation, gravitational-wave propagation, and nonlinear astrophysical systems. Furthermore, the physical origin of the matter-geometry coupling remains largely phenomenological and requires deeper theoretical justification from fundamental field theory or quantum gravity. Additional observational tests using forthcoming cosmological surveys, including Euclid, the Vera C. Rubin Observatory, the Nancy Grace Roman Space Telescope, and future CMB experiments, will provide stronger constraints on the free parameters of $f(Q, L_m)$ gravity and help determine its viability as a fundamental theory of gravitation.

Overall, the reviewed studies demonstrate that $f(Q, L_m)$ gravity constitutes a robust and versatile modified theory capable of explaining multiple stages of cosmic evolution within a single theoretical framework. Its successful application to inflation, late-time accelerated expansion, and the analysis of energy conditions [32, 33, 34] highlights its potential as an alternative description of gravity beyond General Relativity. Continued theoretical developments and increasingly precise observational data will play a crucial role in determining whether this framework can evolve into a comprehensive theory capable of addressing the outstanding problems of modern cosmology.

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